Name: Mantell NCC ID #: N00_____ 24 April 2020

MAT 114-EDA: Foundations of Mathematics for Educators EXAM II Prof. Mantell

Show relevant work (except for Problem 2), answers without support may receive little or no credit. Solutions are due Tuesday, April 28, by 11:59 PM. **Please be certain to sign the statement at the end attesting to your adherence to academic integrity.**

1. Convert the following numbers as indicated.

a) Convert to Roman numerals. (3 points each)

i. 1959 (the year NCC was founded)

$= 1000 + 900 + 50 + 9 \rightarrow$ MCMLIX

ii. 7,780,020,000 (currently the approximate world population)

$= 5{,}000{,}000{,}000 + 2{,}000{,}000{,}000 + 700{,}000{,}000 + 80{,}000{,}000 + 20{,}000 \rightarrow$ $\overline{\overline{\text{V}}}\,\overline{\text{MM}}\;\overline{\text{DCC}}\;\overline{\text{LXXX}}\;\text{XX}$

b) Convert from Roman numerals:

i. MMCDLXXXIV

$\rightarrow 1000 + 1000 + 400 + 50 + 10 + 10 + 10 + 4 =$ 2484

ii. $\overline{\text{MCCCLVMM}}$ (currently the approximate population of Nassau County)

$\rightarrow 1000(1000 + 100 + 100 + 100 + 50 + 5) + 1000 + 1000$

$= 1000 \cdot 1355 + 2000 =$ 1,357,000

c) Convert from the given base to base 10. (5 points each)

i. $3653.2_8 = 3 \cdot 8^3 + 6 \cdot 8^2 + 5 \cdot 8^1 + 3 \cdot 8^0 + 2 \cdot 8^{-1}$

$= 3 \cdot 512 + 6 \cdot 64 + 40 + 3 + \frac{2}{8} =$ 1963.25

ii. 1110110.11_2

$= 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^{-1} + 1 \cdot 2^{-2} =$ 118.75

iii. $10A2F.8_{16}$ $A = 10$, $F = 15$

$= 1 \cdot 16^4 + 10 \cdot 16^2 + 2 \cdot 16^1 + 15 \cdot 16^0 + 8 \cdot 16^{-1}$

$= 65{,}536 + 2560 + 32 + 15 + \frac{8}{16} =$ 68,143.5

d) Convert 1000 from base 10 to base to base 7. (8 points)

$7 \overline{)\,1000}$ R = 6

$7 \overline{)\,142}$ R = 6

$7 \overline{)\,20}$ R = 2

$7 \overline{)\,2}$ R = 6

0 R = 2

2626_7

The Adventure Continues ⟶

2. Name the arithmetic property illustrated by the equations (i.e. Associative, Commutative, or Distributive). (2 points each)

a) $8326+751=751+8326$ → Commutative

b) $3(86-51)=3\cdot86-3\cdot51$ → Distributive

c) $(23\cdot65)\cdot18=23\cdot(65\cdot18)$ → Associative (for multiplication)

3. Determine if the smaller number is a factor of the larger one using the "rules" for divisibility, and briefly explain your answer. (4 points each)

a) 3 and 60,513,540,484,317 → Sum of Digits $=51=3\cdot17$

⇒ Yes!

b) 6 and 60,513,540,484,318 → Sum of Digits $=52$, Not a multiple of 3.
Even, but ---

⇒ No!

c) 11 and 33,709,709,141,177

Alternating Sum $=3-3+7-0+9-7+0-9+1-4+1-1+7-7$
$=-3$, Not a multiple of 11 ⇒ No!

4. Determine the GCF and LCM of the following groups of numbers by using the prime factorization.

a) 168 and 248 (5 points each)

$168=2^3\cdot3\cdot7$

$248=2^3\cdot31$

GCF $=2^3=8$

LCM $=2^3\cdot3\cdot7\cdot31=5208$

b) $2^3\cdot3^2\cdot7\cdot11^2$ and $3^3\cdot5^2\cdot7^2\cdot11$

GCF $=3^2\cdot7\cdot11=693$

LCM $=2^3\cdot3^3\cdot5^2\cdot7^2\cdot11^2=32{,}016{,}600$

5. List the following fractions from smallest to largest by obtaining the LCD: $\frac{4}{7}, \frac{28}{51}, \frac{35}{63}, \frac{104}{189}$. (8 points)

$7=7^1$, $51=3\cdot17$, $63=3^2\cdot7$, $189=3^3\cdot7$

⇒ LCM/LCD $=3^3\cdot7\cdot17=3213$

⇒ $\frac{4}{7}\cdot\frac{3^3\cdot17}{3^3\cdot17}=\frac{1836}{3213}$, $\frac{28}{51}\cdot\frac{3^2\cdot7}{3^2\cdot7}=\frac{1764}{3213}$, $\frac{35}{63}\cdot\frac{3\cdot17}{3\cdot17}=\frac{1785}{3213}$

and $\frac{104}{189}\cdot\frac{17}{17}=\frac{1768}{3213}$

⇒ $\frac{28}{51}, \frac{104}{189}, \frac{35}{63}, \frac{4}{7}$

The Adventure Continues Again ⟶

6. Use the appropriate ratios/proportions to answer the following. (4 points each)

a) Convert 1 year to minutes; knowing how many days/year, how many hours/day, how many minutes/hour, and how many seconds/minute.

$$\frac{1 \text{ year}}{1} \cdot \frac{365 \text{ days}}{1 \text{ year}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} = \boxed{525{,}600 \text{ minutes}}$$

b) Convert 60 miles/hour to feet/second, given 1 mile = 5280 feet and what you know about hours, minutes, and seconds.

$$\frac{60 \text{ miles}}{1 \text{ hour}} \cdot \frac{5280 \text{ ft}}{1 \text{ mile}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} = \boxed{88 \text{ ft/sec}}$$

c) One World Trade Center (a.k.a. The Freedom Tower) has a height of 1792 feet and a square base that is 200 feet on each side. A scale model (i.e. a model with the same proportions) is to be made that is 6 feet tall. What should be the length of the square base of the model (in inches)?

$$\frac{1792 \text{ ft}}{200 \text{ ft}} = \frac{72 \text{ inches}}{x \text{ inches}} \Rightarrow 1792x = 200 \cdot 72$$

$$\Rightarrow \boxed{x \approx 8 \text{ inches}}$$

(6 ft = 72 inches)

7. Convert to $\frac{5}{12}$ a repeating decimal. (5 points)

$$0.416\overline{6}$$

$$12 \overline{)\,5.0000}$$

-48

20

-12

80

-72

8 (Cycle!)

$$\Rightarrow \frac{5}{12} = \boxed{0.41\overline{6}}$$

8. Convert 0.6875 to a fraction with whole numbers in simplest form. (5 points)

$$0.6875 = \frac{6875}{10000} = \frac{25 \cdot 275}{25 \cdot 400} = \frac{275}{400} = \frac{25 \cdot 11}{25 \cdot 16}$$

$$= \boxed{\frac{11}{16}}$$

9. Convert the repeating decimal $0.4\overline{09}$ to a fraction with whole numbers in simplest form. (7 points)

Let $x = 0.4\overline{09} \Rightarrow 10x = 4.\overline{09}$

and $1000x = 409.\overline{09}$

$$-10x = -4.\overline{09}$$

$$990x = 405$$

$$\Rightarrow x = \frac{405}{990} = \frac{45 \cdot 9}{45 \cdot 22}$$

$$= \boxed{\frac{9}{22}}$$

The Adventure Ends For Now! ☹