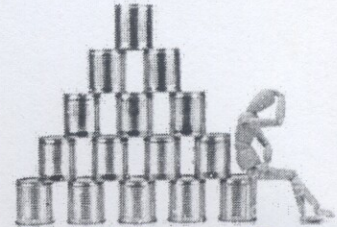


Show relevant work, answers without support may receive little or no credit.

1. a) Imagine a stack of cans in the shape of a triangle where each row has one less can than the row below it, so that there is eventually only one can on top (the image shows this for 5 rows of cans). How many cans would it take to build such a triangular stack with 50 rows? (4 points)
 Do not do this the long way!!!

$$\begin{aligned} \text{I. Let } x &= 1 + 2 + 3 + \dots + 48 + 49 + 50 \\ &+ x = 50 + 49 + 48 + \dots + 3 + 2 + 1 \\ \Rightarrow 2x &= 51 + 51 + 51 + \dots + 51 \\ 2x &= 50 \cdot 51 \Rightarrow x = \frac{50 \cdot 51}{2} = 25 \cdot 51 \\ &= \boxed{1275} \end{aligned}$$



OR II. Use Formula

Determine the following sums.

b) $1234 + 1235 + 1236 + \dots + 2020$

(5 points each)

$$\begin{aligned} \text{Let } x &= 1234 + 1235 + \dots + 2019 + 2020 \\ &+ x = 2020 + 2019 + \dots + 1235 + 1234 \\ \Rightarrow 2x &= 3254 + 3254 + \dots + 3254 + 3254 \leftarrow \text{How many?} \\ &2020 - 1234 + 1 = 787 \\ 2x &= 787 \cdot 3254 \Rightarrow x = \frac{787 \cdot 3254}{2} = \boxed{1,280,449} \end{aligned}$$

c) $3 + 6 + 9 + \dots + 999 = 3(1 + 2 + 3 + \dots + 333)$

$$\begin{aligned} \text{Let } x &= 1 + 2 + 3 + \dots + 331 + 332 + 333 \\ &+ x = 333 + 332 + 331 + \dots + 3 + 2 + 1 \end{aligned}$$

$$\Rightarrow 2x = 334 + 334 + 334 + \dots + 334$$

$$2x = 333 \cdot 334 \Rightarrow x = \frac{333 \cdot 334}{2} = 55,611$$

$$\Rightarrow 3 + 6 + 9 + \dots + 999 = 3 \cdot 55,611 = \boxed{166,833}$$

2. Describe a pattern with the following sequences, then give the next 3 terms. (4 points each)

a) 3, 5, 8, 12, 17, 23, 30, 38.

$$\begin{array}{ccccccc} & \nearrow & \nearrow & \nearrow & \nearrow & \nearrow & \nearrow \\ +2 & +3 & +4 & +5 & +6 & +7 & +8 \end{array}$$

b) 64, 32, 16, 8, 4, 2, 1.

$$\begin{array}{ccccccc} & \nearrow & \nearrow & \nearrow & \nearrow & \nearrow & \nearrow \\ \div 2 & \div 2 & \div 2 & \div 2 & \div 2 & \div 2 & \div 2 \end{array}$$

c) 1, 4, 9, 16, 25, 36, 49, 64.

$$\begin{array}{ccccccc} & \nearrow & \nearrow & \nearrow & \nearrow & \nearrow & \nearrow \\ 1^2 & 2^2 & 3^2 & 4^2 & 5^2 & 6^2 & 7^2 \end{array} \quad \leftarrow 8^2$$

3. The table shows the change in the price of a stock. At the end of the week, the price was \$35.98.
 a) What was the price of the stock at the end of the day on Wednesday? (6 points)

End of Thursday: $\$35.98 - \$0.11 = \$35.87$

End of Wednesday: $\$35.87 + \0.02

$= \boxed{\$35.89}$

Day of the week	Change in price
Monday	↑ \$0.21
Tuesday	↑ \$0.09
Wednesday	↓ \$0.33
Thursday	↓ \$0.02
Friday	↑ \$0.11

- b) What was the price of the stock at the beginning of the week (i.e. Monday morning)?

End of Tuesday: $\$35.89 + \$0.33 = \$36.22$

End of Monday: $\$36.22 - \$0.09 = \$36.13$

Beginning of Monday: $\$36.13 - \0.21

$= \boxed{\$35.92}$

4. p represents the statement: *Today is Wednesday.* q represents: *It is raining.* Express the following symbolic statements in ordinary language. (3 points each)

a) $\sim p \wedge q \Rightarrow$ Today is not Wednesday and it is raining.

b) $p \vee \sim q \Rightarrow$ Today is Wednesday or it is not raining.

c) $\sim q \rightarrow \sim p \Rightarrow$ If it is not raining, then it is not Wednesday.

5. For the statement: *If a Democrat becomes our next president, then many Americans will celebrate.*
 Give the: (3 points each)

a) Converse: If many Americans celebrate, then a Democrat became our next president.

b) Inverse: If a Democrat does not become our next president, then many Americans will not celebrate.

c) Contrapositive: If many Americans do not celebrate, then a Democrat did not become our next president.

- d) Which one is logically equivalent to the original statement?

The Contrapositive is logically equivalent to the original statement.

6. Use a truth table for the following statements to determine which, if any, are logically equivalent. (7 points)

$$p \rightarrow q, \sim(p \wedge \sim q) \text{ and } p \vee \sim q$$

P	q	$p \rightarrow q$	$\sim(p \wedge \sim q)$	$p \vee \sim q$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

↑ Same ↑

$p \rightarrow q$
and
 $\sim(p \wedge \sim q)$
are logically
Equivalent.

7. If p is a True statement, q is False, and r is unknown, then determine the truth value of the following statements (as True, False, or Indeterminate). (3 points each)

a) $p \vee q \Leftrightarrow T \vee F = \boxed{\text{True}}$

b) $\sim q \rightarrow (p \vee r) \Leftrightarrow \sim F \rightarrow (T \vee r)$
 $\Leftrightarrow T \rightarrow T = \boxed{\text{True}}$

c) $(r \vee \sim q) \wedge r \Leftrightarrow (r \vee \sim F) \wedge r$
 $\Leftrightarrow (r \vee T) \wedge r \Leftrightarrow T \wedge r = \boxed{\text{Indeterminate}}$

8. Complete the truth table or the following argument to help determine its validity. (7 points)

$$p \rightarrow q$$

$$\sim p \vee \sim q$$

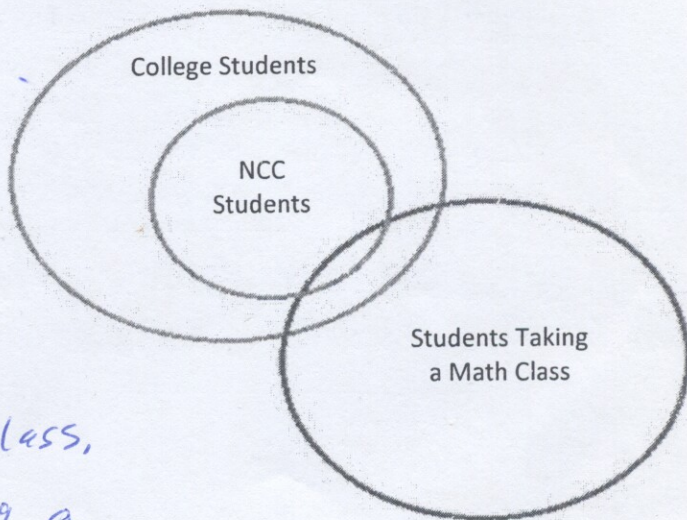
$$\therefore \sim p$$

Valid
Argument

		Premise	Premise	Conclusion
p	q	$p \rightarrow q$	$\sim p \vee \sim q$	$\sim p$
T	T	T	F	F
T	F	F	T	F
F	T	T	T	T
F	F	T	T	T

9. Use quantifiers to write statements that represents the information shown in the diagram. (6 pts)

- ① All NCC Students are College Students.
- ② Some College Students are Taking a Math Class.
- ③ Some NCC Students are Taking a Math Class.
- ④ Some Students Taking a Math Class are not College Students.



10. List the elements of the following sets and give the cardinality. (8 points)

a) $\{x | x \in \mathbb{Z} \wedge x^2 < 100\} = \{-9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, Cardinality = 19

b) $\{x | x \in \mathbb{R} \wedge x^2 = 10\} = \{-\sqrt{10}, \sqrt{10}\}$, Cardinality = 2

c) $\{x | x \in \mathbb{Z} \wedge x^2 = 10\} = \{\}$, i.e. the Empty set, Cardinality = 0.

11. Give a set-builder notation for each set. (5 points)

a) $\{2, 4, 6, 8, \dots, 100\} = \{x | 2 \leq x \leq 100 \wedge x \text{ is even}\}$

b) $\{2, 3, 5, 7, 9, 11, \dots\} = \{x | x = 2 \text{ or } x \text{ is an odd } \# > 2\}$

12. A collegiate tennis team consists of 6 members. Two players must be selected to play doubles. How many different doubles teams are possible? (5 points)

Call the 6 members: A, B, C, D, E, and F

Possible Teams:

AB	BC	CD	DE	EF
AC	BD	CE	DF	
AD	BE	CF		
AE	BF			
AF				

$$5 + 4 + 3 + 2 + 1 = \boxed{15 \text{ possible Teams}}$$

Bonus: A snail is at the bottom of a well which is 30 feet deep. During the day the snail climbs up the wall 3 feet, but slips back down 2 feet at night. How many days does it take the snail to reach the top of the well? Explain! (3 points)

28
Days

Will do in class!