Combinations

e.g. start w/ Permutations.

How many Permutations are possible by taking 3 items from 8 distinct one?

$$\Rightarrow {}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1}$$

(n = 8, r = 3)

$$= 8\cdot7\cdot6 = \boxed{336}$$

e.g. ABC, ACB, BAC, BCA, CAB, CBA

6 permutations

$\Rightarrow$ 336 permutations is counting each combination of 3 items 6 times (i.e. 3!).

$$\Rightarrow {}_8C_3 = \frac{{}_8P_3}{3!} = \frac{336}{6} = \boxed{56}$$

In General: the combination formula for taking "r" items from "n" distinct items.

$$nC_r = \frac{nP_r}{r!}$$

$$\Rightarrow \boxed{nC_r = \frac{n!}{(n-r)!\,r!}}$$

e.g. ① How many different 5-card poker hands are possible?

- Combination
- Without replacement
- $n=52,\ r=5$

$$\Rightarrow {}_{52}C_5 = \frac{52!}{(52-5)!\,5!} = \frac{52!}{47!\,5!}$$

$$= \frac{52\cdot 51\cdot 50\cdot 49\cdot 48\cdot 47\cdot 46\cdots 1}{47\cdot 46\cdots 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$$

$$= \frac{52\cdot 51\cdot 50\cdot 49\cdot 48}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 26\cdot 17\cdot 10\cdot 49\cdot 12$$

$$= \boxed{2,598,960}$$

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② NY LOTTO

$n=59,\ r=6$

- Combination
- Selections are Without replacement

$$\Rightarrow {}_{59}C_6 = \frac{59!}{53!\,6!}$$

$$= \boxed{45,057,474}$$

Different 6 number selections (i.e. Combinations)

(3) Suppose there are a dozen students available who want to serve on a student government committee of size 4.

a) How many different committees can be formed?

- Selections without replacement and order does not matter

⇒ Combination

$$\Rightarrow {}_{12}C_4 = \frac{12!}{(12-4)!4!} = \frac{12!}{8!4!} = 495$$

⇒ 495 Different Committees are possible.

b) Suppose the committee requires officers to be established: President, Vice-Pres., Secretary, and treasurer. Now how many committees can be formed taking into account officers?

- Selections without replacement, but order does matter since positions will be assigned!

⇒ Permutation

$$\Rightarrow {}_{12}P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!} = 11,880$$

⇒ 11,880 Different Committees are possible, with the officer assignments.