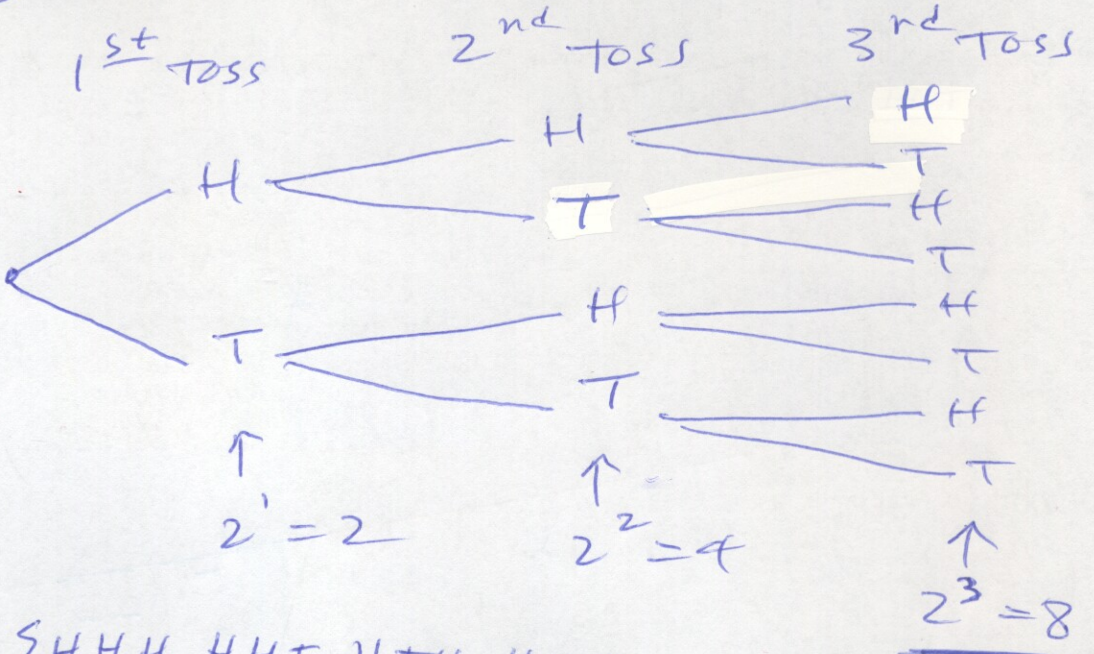
16.2 Counting Techniques

e.g. ① Tossing a Coin 3 Times



$\Rightarrow$ {HHH, HHT, HTH, HTT,
THH, THT, TTH, TTT}

$\Rightarrow$ How many possible outcomes are there for Tossing a fair coin 10 times?

$$2^{10} = \underline{\underline{1024}}$$

$\Rightarrow$ $P(10\ H\text{'s}) = \dfrac{1}{1024}$

$\leftarrow$ # of ways to get 10 H's

$\leftarrow$ total # of possible outcomes

$\Rightarrow$ $P(\text{EXACTLY } 1\ H) = ?$

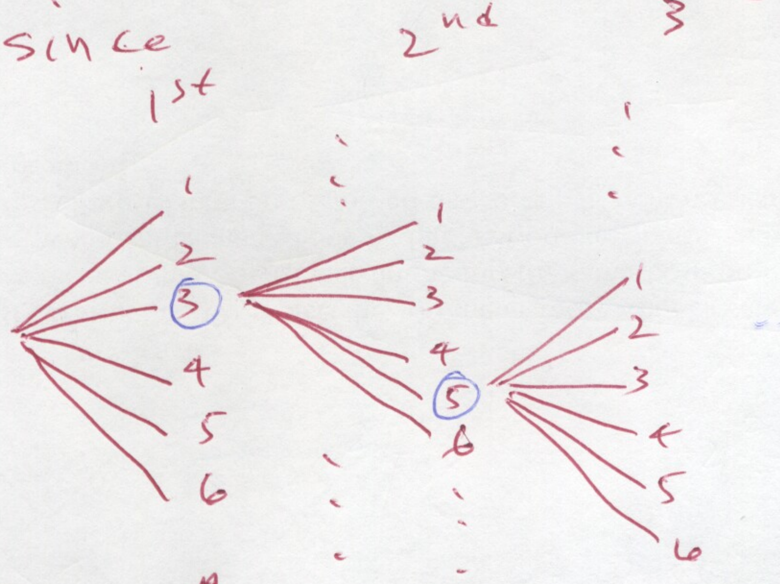
$$= \frac{10}{1024} = \frac{5}{512}$$

$\leftarrow$ HTTTTTTTTT
THTTTTTTTT
⋮
TTTTTTTTTH } 10

(2) How many outcomes (counting order) are possible for rolling 3 ordinary 6-sided dice?

$$6^3 = 216$$

since



6^1 $\quad$ $6^2 = 36$ $\quad$ $6^3 = 216$

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(3) From an ordinary deck of 52 cards, how many ways are there to select* 5 of them (counting order)?

1st	2nd	3rd	4th	5th
52	51	50	49	48

$$\Rightarrow 52 \cdot 51 \cdot 49 \cdot 48 = 311,875,200$$

*Without Replacement

(Permutations) $\Rightarrow$

Permutations vs. Combinations

- Permutations - Selections or Grouping With regard to order.
- Combinations - Selections or Groupings Without regard to order.

* with Replacement

e.g. ① Suppose we take* 2 of the 4 letters: A, B, C, D.

- Permutations:

AB, BA BC, CB CD, DC
AC, CA BD, DB
AD, DA

$\Rightarrow$ $\boxed{12 \text{ permutations}}$

vs.

- Combinations: $\Rightarrow$ $\boxed{6 \text{ Combinations}}$

OR

1^{st}	2^{nd}
4	3

$\Rightarrow 4 \cdot 3 = \boxed{12}$

(2) How many different seating arrangements are possible for 7 students in the front row of a classroom?

1st	2nd	3rd	4th	5th	6th	7th
7	6	5	4	3	2	1

$$\Rightarrow \underbrace{7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}_{7!} = 5040$$

7!, Factorial Notation

$$\Rightarrow n! = n(n-1)(n-2)(n-3)\cdots 1$$

e.g. $4! = 4\cdot 3\cdot 2\cdot 1 = 24$

$8! = 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1$

$= 40{,}320$

(3) Suppose there are 12 students that want to sit in the front row (still 7 chairs). How many different seating arrangements are possible?

1st	2nd	3rd	4th	5th	6th	7th
10	9	8	7	6	5	4

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$

$$\Rightarrow = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{10!}{3!}$$

$$= \frac{10!}{(10-7)!} = \boxed{604,800}$$

Thus, the formula for the # of Permutations of "n" distinct objects taken "r" at a time is:

$${}_nP_r = \frac{n!}{(n-r)!}$$