

Base Conversions Examples

① Base 9 to Base 10

$$\begin{array}{cccc} 3 & 0 & 8 & . & 2 \\ \uparrow & \uparrow & \uparrow & \uparrow & \\ 9^2 & 9^1 & 9^0 & 9^{-1} & \end{array} \quad \begin{aligned} 308.2_9 &= 3 \cdot 9^2 + 0 \cdot 9^1 + 8 \cdot 9^0 + 2 \cdot 9^{-1} \quad (\text{Base } 10) \\ &= 3 \cdot 81 + 0 + 8 \cdot 1 + 2 \cdot \frac{1}{9} \\ &= 243 + 8 + \frac{2}{9} \\ &= \boxed{251 \frac{2}{9}} \quad (\text{Base } 10) \\ &= 251.\bar{2} \end{aligned}$$

Hexadecimal (Base 16)

$$A = 10, B = 11, C = 12$$

$$D = 13, E = 14, F = 15$$

e.g.

Convert:

① 100 in Base 10 to Hex.

Note: $16^0 = 1, 16^1 = 16, 16^2 = 256, 16^3 = 4096$

How many 16^1 "fit" into 100?

$$\begin{array}{r} 6 \quad R=4 \\ 16 \overline{) 100} \end{array}$$

$$\begin{array}{|c|c|} \hline 16^1 = 16 & 16^0 = 1 \\ \hline 6 & 4 \\ \hline \end{array}$$

$$\boxed{64}_{16}$$

$$16 \cdot 6 = 96$$

② Convert 100,000 (Base 10)
to hexadecimal

I. $16^0 = 1, 16^1 = 16, 16^2 = 256$
 $16^3 = 4096, 16^4 = 65,536, 16^5 = 1,048,576$
Too Big!

$$\begin{array}{r} 1 \\ 65,536 \overline{) 100,000} \\ \underline{-65,536} \\ 34,464 \end{array}$$

$$\begin{array}{r} 8 \\ 4096 \overline{) 34464} \\ \underline{-32768} \\ 1696 \end{array}$$

$$\begin{array}{r} 6 \\ 256 \overline{) 1696} \\ \underline{-1536} \\ 160 \end{array}$$

$$\begin{array}{r} 10=A \\ 16 \overline{) 160} \\ \underline{-160} \\ 0 \end{array}$$

$$\begin{array}{r} 0 \\ 1 \overline{) 0} \end{array}$$

⇒

$$\boxed{186A0_{16}}$$

③ Convert $2,345,678_{10}$
to hexadecimal

$$16^0 = 1, 16^1 = 16, 16^2 = 256, 16^3 = 4096,$$

$$16^4 = 65,536, 16^5 = 1,048,576, 16^6 \text{ too big!}$$

"Short-Cut" $[A=10, B=11, C=12, D=13, E=14, F=15]$

$$\begin{array}{r} 146,604 \text{ R} = 14 = E \\ 16 \overline{) 2,345,678} \\ \underline{9162} \end{array}$$

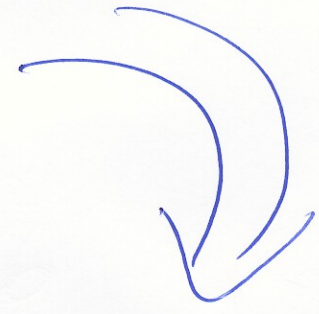
$$\begin{array}{r} 146,604 \text{ R} = 12 = C \\ 16 \overline{) 146,604} \\ \underline{572} \end{array}$$

$$\begin{array}{r} 572 \text{ R} = 10 = A \\ 16 \overline{) 9162} \\ \underline{35} \end{array}$$

$$\begin{array}{r} 35 \text{ R} = 12 = C \\ 16 \overline{) 572} \\ \underline{2} \end{array}$$

$$\begin{array}{r} 2 \text{ R} = 3 \\ 16 \overline{) 35} \\ \underline{0} \end{array}$$

$$\begin{array}{r} 0 \text{ R} = 2 \\ 16 \overline{) 2} \end{array}$$


 $23CACE_{16}$

④

$$\begin{array}{cccccc} A & B & E & D & 2 & \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \\ 16^2 & 16^1 & 16^0 & 16^{-1} & 16^{-2} & \end{array}$$

$$\begin{aligned} &= A \cdot 16^2 + B \cdot 16^1 + E \cdot 16^0 + D \cdot 16^{-1} + 2 \cdot 16^{-2} \\ &= 10 \cdot 256 + 11 \cdot 16 + 14 \cdot 1 + 13 \cdot \frac{1}{16} + \frac{2}{16^2} \\ &= 2560 + 176 + 14 + \frac{13}{16} + \frac{2}{256} \\ &= 2750 + \frac{13}{16} + \frac{1}{128} \end{aligned} \left. \vphantom{\begin{aligned} &= A \cdot 16^2 + B \cdot 16^1 + E \cdot 16^0 + D \cdot 16^{-1} + 2 \cdot 16^{-2} \\ &= 10 \cdot 256 + 11 \cdot 16 + 14 \cdot 1 + 13 \cdot \frac{1}{16} + \frac{2}{16^2} \\ &= 2560 + 176 + 14 + \frac{13}{16} + \frac{2}{256} \\ &= 2750 + \frac{13}{16} + \frac{1}{128} \end{aligned}} \right\} \begin{array}{l} \text{Base} \\ 10 \end{array}$$

$$= \boxed{2750 \frac{105}{128} = 2750.8203125_{10}}$$

Converting a Non-Whole Number in Base 10 to Other Bases - Examples

① 8.5₁₀ to Base 2

$$8_{10} = 2^3 = 1000_2$$

$\begin{matrix} 2^3 & 2^2 & 2^1 & 2^0 \\ \downarrow & \downarrow & \downarrow & \downarrow \end{matrix}$

$$8.5 = 8 + 0.5 \rightarrow 0.5_{10} = 1 \cdot 2^{-1} = 0.1_2$$

$$0.5 = \frac{1}{2} = 2^{-1} \Rightarrow 8.5_{10} = \boxed{1000.1_2}$$

② 8.5₁₀ to Base 6

$$8_{10} = 12_6 \text{ (since } 1 \cdot 6^1 + 2 \cdot 6^0 = 6 + 2 = 8)$$

$$0.5_{10} = \frac{1}{2} = \frac{3}{6} = 3 \cdot 6^{-1} = 0.3_6 \Rightarrow 8.5_{10} = \boxed{12.3_6}$$

③ 489.5₁₀ to Base 7

$$489_{10} = 1266_7$$

How many 7^{-1} "fit into" 0.5? $\Rightarrow \frac{1}{2} \div \frac{1}{7} = \frac{1}{2} \cdot \frac{7}{1} = 3.5$

Now, how many 7^{-2} "fit into" $\frac{1}{14}$? $\Rightarrow \frac{1}{14} \div \frac{1}{7^2}$

$0.5 \div 7^{-1}$
 $\Rightarrow \textcircled{3}$ w/a remainder
 of $\frac{1}{2} - 3 \cdot \frac{1}{7} = \frac{7}{14} - \frac{6}{14} = \frac{1}{14}$

$$\Rightarrow 0.5_{10} = 0.333\dots = 0.\bar{3} = \frac{1}{14} \div \frac{1}{49} = \frac{49}{14} = 3.5, \text{ same as before!}$$

thus, the 3 repeats

$$\Rightarrow 489.5_{10} = \boxed{1266.\bar{3}_7}$$

④ 489.5₁₀ to Base 16

$$489_{10} = 1E9_{16} \text{ and } 0.5_{10} = \frac{1}{2} = \frac{8}{16} = 8 \cdot 16^{-1} = 0.8_{16}$$

$$\Rightarrow 489.5_{10} = \boxed{1E9.8_{16}}$$